Spotlights in Computational Physics and Engineering (SCoPE)

Invited lectures on:

Polytopal discretization and finite element methods for nonlinear structural analysis

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Abstract

The low-order finite element methods that are commonly used in structural analysis often rely on simple element shapes, such as triangles/tetrahedra and quadrilaterals/hexahedra. These consist of a fixed number of nodes, edges and faces per element. From a mesh generation perspective, it is aimed to loosen the geometrical restrictions of classical low-order finite elements and (re)develop new and existing approaches exploiting the benefits that general polytopes have to offer. The common belief that the generalization of finite element formulations towards arbitrary polygons and polyhedra merely increases the computational complexity of the finite element method, is in fact misleading. This lecture series therefore aims to highlight the merits of polytopal discretization and finite element approaches, including the flexibility during mesh generation and refinement, improved higher-order approximation of the field variables in the element interior, and the stabilization of saddle-point problems. Topics covered include polytopal discretization techniques such as Voronoi tessellations in both 2D and 3D, approaches for the construction of linear complete interpolation function on polytopes, and the concept of mesh-dependent inf-sup stabilization of displacement-pressure formulations. The primary focus will be on finite strain applications and hyperelasticity.

This series of lectures will be divided into four 30-minute sessions covering the following topics:

1. Introduction to polytopal discretization methods: Motivation; basics of quadtree and octree meshes for FEM; basics of Voronoi meshes for FEM; properties of Voronoi meshes based on duality; necessity of Lloyd relaxation; mesh degeneracy.
2. Interpolation functions for polytopes: Scaled boundary parameterization and method; macro-element approaches; shape functions based on solution to Poisson's equation; properties of the shape functions; numerical integration; affinity of the isoparametric map; evaluation of efficiency and accuracy.
3. Saddle-point problems: Weak form of the two-field problem; linearization; approximation using polytopes; pressure continuity; volumetric locking tests; assessment of efficiency and accuracy.
4. Mesh topology-based stabilization: Coercivity; inf-sup condition; spurious pressure modes; generalization of the condition for arbitrary low-order elements; assembly over edges/faces; nullspace method; assessment of inf-sup stability using several benchmarks.

When and Where?

- 27.02.2025, 10:00-12:00, Maison du Nombre, MNO 1.030
- 28.02.2025, 10:00-12:00, Maison du Nombre, MNO 1.030

Invitee: Bjorn Sauren*

BJORN SAUREN is a research associate at RWTH Aachen University in Germany since 2020, working on advanced discretization and finite element methods for computational solid mechanics.

Selected Publications

- B. Sauren, S. Klarmann, L. Kobbelt, S. Klinkel. A mixed polygonal finite element formulation for nearly-incompressible finite elasticity. Computer Methods in Applied Mechanics and Engineering (2023)
- B. Sauren, S. Klinkel. On the stability of mixed polygonal finite element formulations in nonlinear analysis. International Journal for Numerical Methods in Engineering (2024)
- B. Sauren, S. Klinkel. Mesh topology-based spurious pressure stabilization in 3D finite elasticity using Voronoi tessellations. Computational Mechanics (2024)
- G. Kikis, B. Sauren, C. Birk, S. Klinkel. Stabilization of topology optimization problems using Voronoi tessellations. PAMM (2024)
- E.T. Ooi, B. Sauren, S. Natarajan, C. Song. An Extensible Set of Parent Elements to Facilitate the Isoparametric Concept for Polygons at Finite Strains: A Scaled Boundary Finite Element Approach. SSRN Preprint (2024)